1. (a) Use the Regular Value Theorem to prove that the set
\[ \left\{ (x, y, z) \in \mathbb{R}^3 \mid \left( \sqrt{x^2 + y^2} - 2 \right)^2 + z^2 = 1 \right\} \]
is an embedded submanifold of \( \mathbb{R}^3 \).
(b) Draw the set. \textit{Hint:} Use cylindrical coordinates \((r, \theta, z)\) on \( \mathbb{R}^3 \).

2. Consider the function
\[ f : \mathbb{R}P^2 \to \mathbb{R} \]
\[ [x^1, x^2, x^3] \mapsto \frac{3(x^1)^2}{(x^1)^2 + (x^2)^2 + (x^3)^2} \]

(a) Find all of the critical points of \( f \). \textit{Hint:} You need to check in all three of our standard coordinate charts \( \{ (U_i, \phi_i) \}_{i=1,2,3} \).
(b) Find the point at which \( f \) takes its maximum value.
(c) Find all values \( c \) for which the Regular Value Theorem implies that \( f^{-1}(c) \) is a manifold.